40. $\int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_0^4 \left(\frac{1}{\sqrt{u}} - 1\right) du$

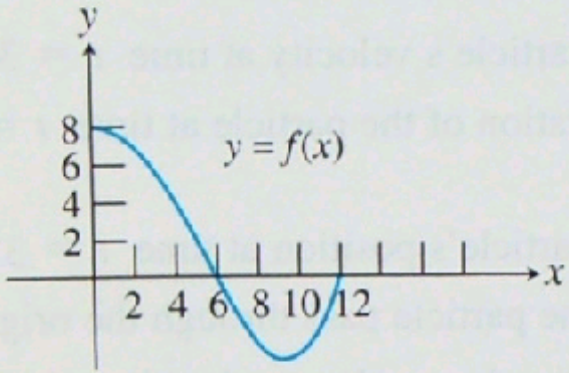
$= \int_0^4 \left(u^{-1/2} - 1\right) du = \frac{x^{1/2}}{\left(\frac{1}{2}\right)} - x$

$= 2x^{1/2} - x = 2\sqrt{x} - x = 0$

57. Let

$$H(x) = \int_0^x f(t)\,dt,$$

where f is the continuous function with domain $[0, 12]$ graphed here.



$H' = f(x)$

H'' = slope of $f(x)$

(Concave up = pos.

(a) Find $H(0)$. $= 0$

(b) On what interval is H increasing? Explain. $[0, 6]$ $[6, 12]$

(c) On what interval is the graph of H concave up? Explain. $[9, 12]$

(d) Is $H(12)$ positive or negative? Explain.

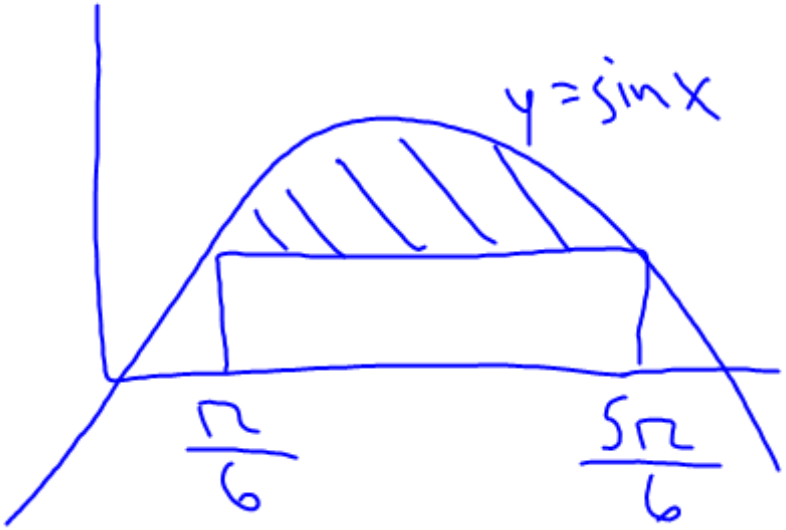
(e) Where does H achieve its maximum value? Explain. $x = 6$

(f) Where does H achieve its minimum value? Explain. $x = 0$

48. $y = \sin x$

$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin x = -\cos x$

$\frac{\pi}{6} \quad = 1.73$



rectangle: $\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{6}\right) = 1.04$

total: 0.685

2004 SCORING GUIDELINES

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

Antideriv. $82x - 8\cos\left(\frac{t}{2}\right)$

Deriv. $4\cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars

$\left(82(30) - 8\cos\left(\frac{30}{2}\right)\right) - \left(82(0) - 8\cos\left(\frac{0}{2}\right)\right)$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(b) $F'(7) = -1.872$ or -1.873

Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

Reason Ans

$2\cos\left(\frac{7}{2}\right) \approx -1.872$

1 : answer with reason

(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min

$\frac{1}{b-a}\int_a^b f(x)\,dx$

3 : { 1 : limits; 1 : integrand; 1 : answer }

(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2

1 : answer

Units of cars/min in (c) and cars/min^2 in (d)

1 : units in (c) and (d)

AP® CALCULUS AB 2002 SCORING GUIDELINES

HW **Question 3**

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?